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Notes on Properties of Holographic Strange Metals

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Abstract

We investigate properties of holographic strange metals in $p+2$-dimensions, generalizing the analysis performed in arXiv:0912.1061. The bulk spacetime is $p+2$-dimensional Lifshitz black hole, while the role of charge carriers is played by probe D-branes. We mainly focus on massless charge carriers, where most of the results can be obtained analytically. We obtain exact results for the free energy and calculate the entropy density, the heat capacity as well as the speed of sound at low temperature. We obtain the DC conductivity and DC Hall conductivity and find that the DC conductivity takes a universal form in the large density limit, while the Hall conductivity is also universal in all dimensions. We also study the resistivity in different limits and clarify the condition for the linear dependence on the temperature, which is a key feature of strange metals. We show that our results for the DC conductivity are consistent with those obtained via Kubo formula and we obtain the charge diffusion constant analytically. The corresponding properties of massive charge carriers are also discussed in brief.

Contents

1 Introduction

One of the most interesting subjects in condensed matter physics is to understand the properties of strongly interacting systems of fermions, in particular, to understand the thermodynamic and transport properties of the “strange metal” phases of heavy fermion

compounds [1] and high temperature superconductors [2, 3]. The “strange metal” phases possess various interesting “non-Fermi liquid” features. For example, the DC resistivity is linear in temperature T when T is much less than the chemical potential μ [4]. On the other hand, the AC conductivity exhibits a non-trivial scaling behavior $\sigma(\omega) \sim \omega^{-\nu}$ with $\nu \neq 1$ [5]. Furthermore, the Hall conductivity also becomes anomalous [6]. It is widely believed that to gain a better understanding of such properties requires us to go beyond the regime of weak coupling.

Recently, inspired by the AdS/CFT correspondence [7, 8], investigations on the applications of AdS/CFT to condensed matter physics (also named as the AdS/CMT correspondence) have been accelerated enormously [9]. Since the AdS/CFT correspondence provides us with useful tools for understanding the dynamics of strongly coupled field theories in the dual weakly coupled gravity side, it may open a new window for studying real-world physics in the context of holography. Thus one can expect that the peculiar properties of “strange metals” can be well understood via the AdS/CFT correspondence.

One class of such holographic models of strange metals were proposed in [10, 11, 12, 13, 14]. In such models the dual bulk spacetime was described by an extremal RN-AdS$_4$ black hole, whose near horizon geometry contains an AdS$_2$ part. It was shown that the low energy behavior of these non-Fermi liquids was controlled by the near horizon IR fixed point. Examples whose single-particle spectral function and transport behavior resembling those of strange metals were found within this class of models.

Recently a complementary approach was proposed in [15], where they considered a bulk gravitational background which was dual to a neutral Lifshitz-invariant quantum critical theory, while the gapped probe charge carriers were described by D-branes. The non-Fermi liquid scalings, such as linear resistivity, observed in strange metal regimes can be realized by choosing the dynamical critical exponent z appropriately. They also outlined three distinct string theory realizations of Lifshitz geometries. Similar investigations on charged dilaton black holes were carried out in [16, 17], where strange metallic behavior was found by properly fixing the parameters of the bulk gravity theory.

In this paper we will study properties of holographic strange metals by employing the approach proposed in [15]. The bulk theory is chosen to be a $p+2$-dimensional asymptotically Lifshitz black hole, while the charge carriers are still represented by probe Dq-branes. For a practical application in condensed matter physics, the value of p is fixed.

For example, one should set $p = 2$ to study the $(2+1)$-dimensional layered systems. However, our physical spacetime may be higher dimensional with extra dimensions, and more spacetime dimensions might be holographically generated when extra adjoint fields are involved. Therefore, it is of interest to consider generalizations to various dimensions and try to find the universal behavior.

We shall consider both massless and massive charge carriers. In the language of D-brane physics, the massive case is related to a non-trivial embedding profile of the probe D-brane into bulk spacetime. We mainly focus on the calculations of massless charge carriers and we will give some remarks on the massive case. We find that for massless charge carriers, the chemical potential μ and the free energy Ω can be obtained analytically. Therefore one can work out other thermodynamic quantities by standard procedures. It turns out that at low temperature, the specific heat behaves like $c_v \propto T^{2p/z}/d$, where z is the dynamical component and d is related to the charge density. It can resemble a gas of free bosons, fermions or new types of holographic quantum liquids by fixing the values of z and p. The speed of sound at low temperature is given by z/p. We calculate the DC conductivity and DC Hall conductivity by introducing corresponding $U(1)$ gauge fields on the worldvolume of the probe D-branes. In the large charge density limit, the expression for the DC conductivity is the same as the four-dimensional counterpart, while it becomes p-dependent in the vanishing charge density limit. On the contrary, the DC Hall conductivity exhibits universal behavior in general $p+2$-dimensions. Therefore the linear dependence of temperature T for the resistivity, which is a common feature of strange metals, can be realized by adjusting parameters in the theory. We re-obtain the DC conductivity by using the result derived from Kubo formula, which proves consistency of the two approaches. We also obtain analytic results for the charge diffusion constant of massless charge carriers by using a generalized version of the formula derived from the membrane paradigm. The properties of massive charge carriers are also discussed in brief.

The rest of the paper is organized as follows. The general setup will be illustrated in section 2 and the thermodynamics of massless charge carriers will be discussed in section 3. In section 4 we will calculate the DC conductivity and Hall conductivity in general $p+2$-dimensions and discuss various limits of the resistivity . We will re-obtain the DC conductivity by Kubo formula and the charge diffusion constant in section 5. Remarks on massive charge carriers are given in section 6. Finally, summary and discussion are

presented in section 7.

2 The Setup

Consider the following ten-dimensional bulk spacetime,

$$ds^2 = L^2[-r^{2z}f(r)dt^2 + \frac{dr^2}{r^2 f(r)} + r^2 d\vec{x}_p^2] + L^2 d\Omega_{8-p}^2, \tag{2.1}$$

where the metric of S^{8-p} is given by

$$d\Omega_{8-p}^2 = d\theta^2 + \cos^2\theta d\Omega_n^2 + \sin^2\theta d\Omega_{7-p-n}^2. \tag{2.2}$$

The non-spherical part of this metric is a $p+2$-dimensional asymptotically Lifshitz black hole with $p \geq 2$, whose temperature is given by

$$T = \frac{r_+^{z+1}}{4\pi}\left|f'(r_+)\right| \equiv \frac{\lambda r_+^z}{4\pi z}, \tag{2.3}$$

where r_+ denotes the radial position of the horizon. The parameter λ is determined by the explicit form of the function $f(r)$.

When $f(r) = 1$, this part of the metric possesses the following anisotropic scaling symmetry,

$$t \;\rightarrow\; \lambda^z t, \quad \vec{x} \;\rightarrow\; \lambda\vec{x}, \quad r \;\rightarrow\; \frac{r}{\lambda}. \tag{2.4}$$

Such backgrounds can be considered as the gravity duals of Lifshitz fixed points, which was initially constructed in [18]. Asymptotically Lifshitz black hole solutions were investigated in [19] and [20]. The relations between Lifshitz black holes and heavy fermion metals were discussed in [21]. It should be emphasized that some of the analytic solutions involve a non-trivial dilaton even at zero temperature, which breaks the scaling symmetry of Lifshitz spacetime. Lifshitz spacetimes in string theory with unconventional scaling symmetry were extensively studied in [22]. Since three distinct approaches for realizing Lifshitz geometries in string theory were proposed in [15], we may make the above mentioned ansatz in ten-dimensional spacetime and consider D-brane probes in such backgrounds.

We assume that the probe Dq-brane extends all the spacial directions and wraps the S^n part inside S^{8-p}, which leads to $q = p+n+1$. The dynamics of the Dq-brane is described by the following DBI action (neglecting the Wess-Zumino term)

$$S_{DBI} = -T_q \int dt dr d^p x d\Omega_n e^{-\phi}\sqrt{-\det(g_{ab} + 2\pi\alpha' F_{ab})}, \tag{2.5}$$

where T_q denotes the tension of the Dq-brane, $T_q = 1/((2\pi)^p g_s \ell_s^{p+1})$. g_{ab} and F_{ab} are the induced metric and the $U(1)$ gauge field strength on the worldvolume of the Dq-brane. The scaling symmetry of Lifshitz geometry requires that the dilaton should be constant, $\phi = \phi_0$, thus we can arrive at the effective action

$$S_q = -\tau_{\text{eff}} \int dt dr d^p x \sqrt{-\det(g_{ab} + 2\pi\alpha' F_{ab})}, \tag{2.6}$$

where τ_{eff} is given by

$$\tau_{\text{eff}} = T_q L^n \text{Vol}(S^n) e^{-\phi_0}.$$

However, as pointed out in [15], incorporating a non-trivial dilaton might be helpful to realistic model-building. So here we will also discuss the situation with non-trivial dilaton, whose effective action turns out to be

$$\tilde{S}_q = -\tilde{\tau}_{\text{eff}} \int dt dr d^p x e^{-\phi} \sqrt{-\det(g_{ab} + 2\pi\alpha' F_{ab})}, \tag{2.7}$$

where

$$\tilde{\tau}_{\text{eff}} = T_q L^n \text{Vol}(S^n).$$

In the following sections we will mainly focus on the details for massless charge carriers, both with trivial and non-trivial dilaton. Remarks on the corresponding properties for massive charge carriers will be given in section 6. For simplicity, we will set $L = 1$ in the following and restore the factor of L in the final results for the physical quantities, such as the conductivity, by dimensional analysis.

3 Thermodynamics of massless charge carriers

We discuss the thermodynamics of massless charge carriers at finite charge density in this section. As claimed in [23], once a non-vanishing worldvolume gauge field strength F_{0r} is turned on, only black hole embeddings are physically allowed. That is, the internal S^n of the probe Dq-brane never collapse to zero volume and the Dq-brane extends to, and intersects the horizon. However, in [24, 25] it was argued that Minkowski embeddings were physical and should be considered, due to the fact that the black hole configurations cannot realize chemical potential below a critical value. Later, this problem was resolved in [26] by including Minkowski branes with a constant A_0.

In this section we will divide both sides of (2.6) and (2.7) by the volume of $\mathbb{R}^{1,p}$, working with action densities. We also turn on the time component of the worldvolume gauge field A_0 and absorb the factor of $2\pi\alpha'$ into A_0 for simplicity. Then the DBI action can be expressed as

$$S_q = -\tau_{\text{eff}} \int dr r^p \sqrt{r^{2z-2} - A_0'^2}. \tag{3.1}$$

By defining the charge density

$$\rho = \frac{\delta \mathcal{L}}{\delta A_0'} = \frac{\tau_{\text{eff}} r^p A_0'}{\sqrt{r^{2z-2} - A_0'^2}}, \tag{3.2}$$

the solution for A_0' is given by

$$A_0' = \frac{d r^{z-1}}{\sqrt{r^{2p} + d^2}}, \tag{3.3}$$

where $d = \rho/\tau_{\text{eff}}$.

In the grand-canonical ensemble, the free energy density Ω is given by minus the on-shell value of the action. After plugging in the solution for A_0' we obtain

$$\Omega = \tau_{\text{eff}} \int_{r_+}^{\infty} \frac{r^{z-1+2p}}{\sqrt{r^{2p} + d^2}} dr. \tag{3.4}$$

The divergence can be regulated by background subtraction or local counterterms. The chemical potential μ in the grand-canonical ensemble is determined by $A_0(\infty)$. Notice that $A_0(r_+) = 0$, so the chemical potential is given by

$$\mu = \int_{r_+}^{\infty} A_0' dr. \tag{3.5}$$

It can be seen that our expressions above reduce to those derived in [27] when $z = 1$.

It was observed in [26] that such integrals can be worked out in terms of Beta functions and incomplete Beta functions, whose definitions are given as follows

$$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} = \int_0^1 dt (1-t)^{b-1} t^{a-1} = \int_0^{\infty} du (1+u)^{-(a+b)} u^{a-1}, \tag{3.6}$$

$$B(x;a,b) = \int_0^x (1-t)^{b-1} t^{a-1} = \int_0^{x/(1-x)} du (1+u)^{-(a+b)} u^{a-1}. \tag{3.7}$$

Therefore one can arrive at the analytic expressions for the free energy and the chemical potential

$$\mu = \mu_0 - \frac{r_+^z}{z} {}_2F_1[\frac{z}{2p}, \frac{1}{2}; 1 + \frac{z}{2p}; -\frac{r_+^{2p}}{d^2}], \tag{3.8}$$

$$\Omega = \Omega_0 - \frac{\tau_{\rm eff} r_+^{2p+z}}{(2p+z)d} {}_2F_1[1+\frac{z}{2p}, \frac{1}{2}; 2+\frac{z}{2p}; -\frac{r_+^{2p}}{d^2}] + \Omega_{\rm ct}, \tag{3.9}$$

where the following formulae have been used

$$B(x;a,b) = a^{-1}x^a {}_2F_1[a, 1-b; a+1; x], \tag{3.10}$$

$${}_2F_1[a,b;c;x] = (1-x)^{-a} {}_2F_1[a, c-b; c; \frac{x}{x-1}]. \tag{3.11}$$

The parameters μ_0 and Ω_0 in (3.8) and (3.9) are zero-temperature values, given by

$$\mu_0 = d^{z/p}\alpha(p), \quad \alpha(p) = \frac{1}{2z}B(1+\frac{z}{2p}, \frac{1}{2} - \frac{z}{2p}), \tag{3.12}$$

$$\Omega_0 = -\frac{z\tau_{\rm eff}}{(z+p)\alpha(p)^{p/z}} \mu_0^{1+p/z}. \tag{3.13}$$

One can check that these results are in agreement with those obtained in [28] when $z = 1$.

Next we calculate the thermodynamic quantities in the grand-canonical ensemble. It should be pointed out that the term $\Omega_{\rm ct}$ in (3.9) stands for the contribution from the counterterms in the spirit of holographic renormalization. As was observed in [28], $\Omega_{\rm ct}$ is independent of the density. Since we do not have a well-established holographic renormalization scheme for probe D-branes in Lifshitz backgrounds, we assume that here the contribution from the counterterms is still independent of the density. We shall focus on the density-dependent part of the free energy $\Delta\Omega \equiv \Omega - \Omega_{\rm ct}$. Notice that at low temperature, both (3.8) and (3.9) can be expanded as series in $T/\mu_0 \ll 1$. The charge density is

$$\rho = -\frac{\partial \Delta\Omega}{\partial \mu} = \tau_{\rm eff} d, \tag{3.14}$$

which provides a consistency check. One then computes the entropy density $s(\mu, T)$ in the grand-canonical ensemble

$$s(\mu, T) = -\left(\frac{\partial \Delta\Omega}{\partial T}\right)_\mu = \frac{4\pi\rho}{\lambda} + \frac{\tau_{\rm eff}}{2zd}\left(\frac{4\pi z}{\lambda}\right)^{1+2p/z} T^{2p/z}, \tag{3.15}$$

where the first term gives the entropy density at zero temperature. Finally, the specific heat at low temperature is given by

$$c_V = T\left(\frac{\partial s}{\partial T}\right)_\rho = \frac{\tau_{\rm eff} p}{z^2 d}\left(\frac{4\pi z}{\lambda}\right)^{1+2p/z} T^{2p/z}. \tag{3.16}$$

One can find agreement with the results obtained in [28] once again in the $z = 1$ case.

One may compare the specific heat c_V in (3.16) to that of a gas of free bosons or fermions. As is well known, the low-temperature specific heat for a gas of free bosons is proportional to T^p, while the low-temperature specific heat for a gas fermions is proportional to T for any p. Therefore our result indicates that when $z = 2$, the behavior of the specific heat looks like a gas of free bosons. On the other hand, it resembles a gas of fermions when $z = 2p$. Finally, it may be regarded as a new type of quantum liquid in other cases.

Another interesting quantity is the speed of sound at low temperature. In this limit the pressure is given by

$$P = -\Omega_0 = \frac{z\tau_{\rm eff}}{(z+p)\alpha(p)^{p/z}}\mu_0^{1+p/z}. \tag{3.17}$$

The energy density is

$$\epsilon = \Omega_0 + \mu_0\rho = \frac{p\tau_{\rm eff}}{(z+p)\alpha(p)^{p/z}}\mu_0^{1+p/z}. \tag{3.18}$$

One can easily find that $\epsilon = (p/z)P$. Thus the speed of sound is determined by

$$c_s^2 = \frac{dP}{d\epsilon} = \frac{z}{p}. \tag{3.19}$$

Note that an upper bound on the speed of sound was proposed in [29, 30], which gives $c_s^2 \leq 1/3$ in five-dimensional bulk spacetime. Then such a bound might be violated in five dimensions, i.e. $p = 3$, when $z > 1$. However, it is believed that such a bound always holds at high temperature. Therefore the violation of this bound with $z > 1$ may not be seen as a contradiction.

When a non-trivial dilaton field, which behaves as $e^{-\phi} \propto r^\kappa$, is incorporated, the charge density, chemical potential and on-shell action turn out to be

$$\tilde{\rho} = \frac{\delta\mathcal{L}}{\delta A_0'} \sim \frac{\tilde{\tau}_{\rm eff} r^{p+\kappa} A_0'}{\sqrt{r^{2z-2} - A_0'^2}}, \tag{3.20}$$

$$\tilde{\mu} = \int_{r_+}^{\infty} A_0' dr \sim \int_{r_+}^{\infty} \frac{\tilde{d} r^{z-1}}{\sqrt{r^{2(p+\kappa)} + \tilde{d}^2}} dr, \tag{3.21}$$

$$\tilde{\Omega} \sim \tilde{\tau}_{\rm eff} \int_{r_+}^{\infty} \frac{r^{z-1+2(p+\kappa)}}{\sqrt{r^{2(p+\kappa)} + \tilde{d}^2}} dr, \tag{3.22}$$

where $\tilde{\tau}_{\rm eff} = T_q L^n {\rm Vol}(S^n)$ and $\tilde{d} = \tilde{\rho}/\tilde{\tau}_{\rm eff}$. It can be seen that all the thermodynamic quantities in the presence of a non-trivial dilaton can be obtained by simply replacing p by $p+\kappa$ in the expressions for those with a constant dilaton.

In particular, the specific heat becomes

$$\tilde{c}_V \sim T^{2(p+\kappa)/z}. \tag{3.23}$$

Therefore the specific heat scales as a gas of free bosons when $z = 2(p+\kappa)/p$ and as a gas of fermions when $z = 2(p+\kappa)$. It may be regarded as a new type of quantum liquid for other cases. The speed of sound is given by

$$\tilde{c}_s^2 = \frac{z}{p+\kappa}. \tag{3.24}$$

Thus the upper bound proposed in [29, 30] can be saturated by setting $z = (p+\kappa)/3$ in five dimensions.

4 Conductivity and resistivity

We calculate the DC conductivity, DC Hall conductivity and resistivity in this section, following the method proposed in [31] and [32]. We find that in the large charge density limit, the DC conductivity exhibits certain universal behavior, independent of the spacetime dimension. On the other hand, the DC Hall conductivity always takes the same form as the four-dimensional counterpart. Finally, we obtain the resistivity in different limits and figure out the requirements of linear dependence on the temperature T.

4.1 DC conductivity

Generally speaking, there are two different ways for calculating DC conductivity in flavor brane systems. One is the traditional way, that is, to obtain the conductivity via Kubo formula. The other elegant way, which was proposed in [31], is more efficient for flavor brane systems. The main strategy is to turn on an electric field $E \equiv F_{tx}$ on the D-brane probe and compute the corresponding current J^x in the boundary theory. Then the conductivity can be read off from Ohm's law $J^x = \sigma E$.

We will calculate the DC conductivity by applying the second method in this section and present discussions on the first method in the next section. To make comparison with [15], we take the following coordinate transformation before performing the calculations

$$v = \frac{1}{r}, \quad v_+ = \frac{1}{r_+}. \tag{4.1}$$

Then the metric of the $(p+2)$-dimensional Lifshitz black hole becomes(setting $L = 1$)

$$ds^2 = -\frac{f(v)}{v^{2z}}dt^2 + \frac{dv^2}{v^2 f(v)} + \frac{1}{v^2}d\vec{x}_p^2 \tag{4.2}$$

and the temperature is given by

$$T = \frac{\lambda}{4\pi z v_+^z}. \tag{4.3}$$

After taking the ansatz for the worldvolume gauge field

$$A = A_0(v)dt + (-Et + h(v))dx, \tag{4.4}$$

the effective action (2.6) becomes

$$S_q = -\tau_{\text{eff}} \int dt dv d^p x g_{xx}^{\frac{p-1}{2}} \sqrt{-g_{tt}g_{vv}g_{xx} - (2\pi\alpha')^2(g_{vv}E^2 + g_{xx}A_0'^2 + g_{tt}h'^2)}. \tag{4.5}$$

Note that the action contains only A_0' and $h'(v)$, which leads to two conserved quantities

$$C = -\frac{g_{xx}^{\frac{p+1}{2}} A_0'}{\sqrt{-g_{tt}g_{vv}g_{xx} - (2\pi\alpha')^2(g_{vv}E^2 + g_{xx}A_0'^2 + g_{tt}h'^2)}}, \tag{4.6}$$

$$H = -\frac{g_{xx}^{\frac{p-1}{2}} g_{tt}h'}{\sqrt{-g_{tt}g_{vv}g_{xx} - (2\pi\alpha')^2(g_{vv}E^2 + g_{xx}A_0'^2 + g_{tt}h'^2)}}. \tag{4.7}$$

It can be observed that

$$g_{tt}h'C = g_{xx}A_0'H. \tag{4.8}$$

By combining (4.6), (4.7) and (4.8), we can arrive at the following expression for A_0'

$$A_0'^2 = -\frac{C^2 g_{tt}g_{vv}}{g_{xx}} \frac{g_{tt}g_{xx} + (2\pi\alpha'^2 E^2)}{g_{tt}g_{xx}^p + (2\pi\alpha')^2(C^2 g_{tt} + H^2 g_{xx})}, \tag{4.9}$$

which results in the asymptotic behavior for A_0 near the boundary $v \to 0$

$$A_0 = \mu - \frac{C}{z-p}\frac{1}{v^{z-p}} + \cdots, \quad \text{for } z \neq p, \tag{4.10}$$

or

$$A_0 = \mu + C \log \frac{v}{\Lambda} + \cdots , \quad \text{for } z = p, \tag{4.11}$$

where μ denotes the chemical potential and Λ stands for the UV cutoff. In the meantime, the asymptotic behavior of $h(v)$ is given by

$$h(v) = h_0 + \frac{H}{(z+p-2)} v^{z+p-2} + \cdots , \tag{4.12}$$

where we can set $h_0 = 0$.

After substituting (4.8) and (4.9) back into (4.5), the effective action becomes

$$S_q = -\tau_{\text{eff}} \int dt dv d^p x g_{xx}^{\frac{2p-1}{2}} \sqrt{-g_{tt} g_{vv}} \sqrt{\frac{g_{tt} g_{xx} + (2\pi\alpha')^2 E^2}{g_{tt} g_{xx}^p + (2\pi\alpha')^2 (C^2 g_{tt} + H^2 g_{xx})}}. \tag{4.13}$$

As pointed out in [31], both the numerator and the denominator of the term inside the square root of (4.13) change sign between the horizon $v = v_+$ and the boundary $v = 0$. Therefore in order to ensure the reality of the on-shell action, the sign change must appear at the same radial position $0 < v_* < v_+$,

$$g_{tt} g_{xx} + (2\pi\alpha')^2 E^2 \big|_{v=v_*} = 0, \tag{4.14}$$

$$g_{tt} g_{xx}^p + (2\pi\alpha')^2 (C^2 g_{tt} + H^2 g_{xx}) \big|_{v=v_*} = 0. \tag{4.15}$$

From (4.14) we can obtain

$$f(v_*) = (2\pi\alpha')^2 E^2 v_*^{2z+2}. \tag{4.16}$$

Finally, by substituting (4.16) into (4.15) and making the following identifications for the currents

$$J^t = (2\pi\alpha')^2 \tau_{\text{eff}} C, \quad J^x = (2\pi\alpha')^2 \tau_{\text{eff}} H, \tag{4.17}$$

we can read off the conductivity

$$\sigma = \sqrt{(2\pi\alpha')^4 \tau_{\text{eff}}^2 (\frac{L}{v_*})^{2p-4} + (\frac{2\pi\alpha'}{L^2})^2 (J^t)^2 v_*^4} \tag{4.18}$$

from Ohm's law, where we have restored the factor of L by dimensional analysis. Notice that the right hand side of (4.18) is still written as the mean square root of two terms, which is similar to the result for general Dp/Dq systems in [31]. The first term may

be interpreted as contribution from thermally produced pairs of charge carriers, which is expected to be Boltzmann suppressed when the mass of the charge carriers becomes large. The second term is independent of the spacetime dimension. Furthermore, (4.18) reduces to the four-dimensional result obtained in [15] when $p = 2$.

When a non-trivial dilaton is introduced, the effective action becomes

$$\tilde{S}_q = -\tilde{\tau}_{\rm eff} \int dt dv d^p x e^{-\phi} g_{xx}^{\frac{p-1}{2}} \sqrt{-g_{tt} g_{vv} g_{xx} - (2\pi\alpha')^2 (g_{vv} E^2 + g_{xx} A_0'^2 + g_{tt} h'^2)}. \tag{4.19}$$

The corresponding two conserved quantities are given by

$$\tilde{C} = -\frac{e^{-\phi} g_{xx}^{\frac{p+1}{2}} A_0'}{\sqrt{-g_{tt} g_{vv} g_{xx} - (2\pi\alpha')^2 (g_{vv} E^2 + g_{xx} A_0'^2 + g_{tt} h'^2)}}, \tag{4.20}$$

$$\tilde{H} = -\frac{e^{-\phi} g_{xx}^{\frac{p-1}{2}} g_{tt} h'}{\sqrt{-g_{tt} g_{vv} g_{xx} - (2\pi\alpha')^2 (g_{vv} E^2 + g_{xx} A_0'^2 + g_{tt} h'^2)}}. \tag{4.21}$$

Now the on-shell action turns out to be

$$\tilde{S}_q = -\tilde{\tau}_{\rm eff} \int dt dv d^p x e^{-2\phi} g_{xx}^{\frac{2p-1}{2}} \sqrt{-g_{tt} g_{vv}} \sqrt{\frac{g_{tt} g_{xx} + (2\pi\alpha')^2 E^2}{e^{-2\phi} g_{tt} g_{xx}^p + (2\pi\alpha')^2 (C^2 g_{tt} + H^2 g_{xx})}}. \tag{4.22}$$

Finally we can obtain the DC conductivity in a similar way

$$\tilde{\sigma} = \sqrt{(2\pi\alpha')^4 e^{-2\phi_*} \tilde{\tau}_{\rm eff}^2 (\frac{L}{v_*})^{2p-4} + (\frac{2\pi\alpha'}{L^2})^2 (J^t)^2 v_*^4}, \tag{4.23}$$

which reduces to (4.18) in the limit of $\phi = \phi_0$.

4.2 DC Hall conductivity

One can also calculate the DC Hall conductivity by applying the techniques of [31]. For general Dp/Dq systems such analysis was carried out in [32] and the Hall conductivity in four-dimensional Lifshitz background was obtained in [15]. Here we extend the analysis to general $p+2$-dimensional spacetime and we will see that the Hall conductivity exhibits universal behavior.

Once we introduce a constant magnetic field on the worldvolume of the probe D-brane, the ansatz for the $U(1)$ gauge field becomes

$$A_0 = A_0(v), \quad A_x(v,t) = -Et + f_x(v), \quad A_y(v,x) = Bx + f_y(v). \tag{4.24}$$

Then the effective action turns out to be

$$\begin{aligned} S_q &= -\tau_{\rm eff}\int dtdvd^px\sqrt{-\det(g_{ab}+2\pi\alpha' F_{ab})} \\ &\equiv -\tau_{\rm eff}\int dtdvd^px g_{xx}^{\frac{p-2}{2}}\sqrt{GF}, \end{aligned} \tag{4.25}$$

where

$$\begin{aligned} GF &= -g_{tt}g_{vv}g_{xx}^2-(2\pi\alpha')^2g_{xx}^2A_0'^2-(2\pi\alpha')^2g_{vv}g_{xx}E^2 \\ &\quad -(2\pi\alpha')^2g_{tt}g_{vv}B^2-(2\pi\alpha')^2g_{tt}g_{xx}(f_x'^2+f_y'^2) \\ &\quad -(2\pi\alpha')^4A_0'^2B^2-(2\pi\alpha')^4f_y'^2E^2+2(2\pi\alpha')^4A_0'f_y'EB. \end{aligned} \tag{4.26}$$

Since the action depends on the derivatives A_0', f_x' and f_y', we can obtain the following conserved quantities

$$[-g_{xx}^2A_0'-(2\pi\alpha')^2B^2A_0'+(2\pi\alpha')^2EBf_y'](\sqrt{GF})^{-1}g_{xx}^{\frac{p-2}{2}}=C, \tag{4.27}$$

$$-g_{tt}g_{xx}f_x'(\sqrt{GF})^{-1}g_{xx}^{\frac{p-2}{2}}=H, \tag{4.28}$$

$$[-g_{tt}g_{xx}f_y'-(2\pi\alpha')^2E^2f_y'+(2\pi\alpha')^2EBA_0'](\sqrt{GF})^{-1}g_{xx}^{\frac{p-2}{2}}=M. \tag{4.29}$$

After solving for A_0', f_x' and f_y' from the above equations and plugging the solutions back into the effective action, we obtain

$$S_q=\tau_{\rm eff}\int dvg_{xx}^{p-1}\sqrt{-g_{tt}g_{vv}}\frac{\xi}{\sqrt{\xi\eta-a^2}}, \tag{4.30}$$

where

$$\xi=-[(2\pi\alpha')^2E^2g_{xx}+(2\pi\alpha')^2B^2g_{tt}+g_{tt}g_{xx}^2], \tag{4.31}$$

$$\eta=-g_{tt}g_{xx}^p-(2\pi\alpha')^2[g_{tt}C^2+g_{xx}(H^2+M^2)], \tag{4.32}$$

$$a=(2\pi\alpha')^2(MEg_{xx}-BCg_{tt}). \tag{4.33}$$

As pointed out in [32], to ensure the reality of the effective action, ξ,η and a must share the same zero v_*. From (4.31) we obtain

$$f(v_*)=\frac{(2\pi\alpha')^2E^2v_*^{2z+2}}{1+(2\pi\alpha')^2B^2v_*^4}. \tag{4.34}$$

Then we can substitute (4.34) into (4.32) and (4.33) to solve for H and M, and make the following identifications for the currents

$$J^t = (2\pi\alpha')^2\tau_{\rm eff}C, \quad J^x = (2\pi\alpha')^2\tau_{\rm eff}H, \quad J^y = (2\pi\alpha')^2\tau_{\rm eff}M. \tag{4.35}$$

Finally, the conductivity tensor $J^i = \sigma^{ij}E_j$ are given by

$$\sigma^{xx} = \frac{1}{1+(\frac{2\pi\alpha'}{L^2})^2B^2v_*^4}\sqrt{(2\pi\alpha')^4\tau_{\rm eff}^2(\frac{L}{v_*})^{2p-4}(1+(\frac{2\pi\alpha'}{L^2})^2B^2v_*^4)+(\frac{2\pi\alpha'}{L^2})^2(J^t)^2v_*^4}, \tag{4.36}$$

$$\sigma^{xy} = \frac{BJ^tv_*^4}{(\frac{L^2}{2\pi\alpha'})^2+B^2v_*^4}. \tag{4.37}$$

Here are several remarks on the conductivity tensor

- σ^{xx} reduces to the expression obtained in previous section when $B=0$.
- σ^{xy} does not depend on the dimension of spacetime, which is the same as that for general Dp/Dq systems analyzed in [32].
- One interesting quantity for strange metals is the ratio σ^{xx}/σ^{xy}. It has been known that the ratio scales as $\sigma^{xx}/\sigma^{xy}\sim T^2$ for strange metals, while $\sigma^{xx}/\sigma^{xy}\sim 1/\sigma^{xx}$ in Drude theory. It can be easily seen that

$$\frac{\sigma^{xx}}{\sigma^{xy}} = (\frac{L^2}{2\pi\alpha'})^2\frac{1}{J^tBv_*^4}\sqrt{(2\pi\alpha')^4\tau_{\rm eff}^2(\frac{L}{v_*})^{2p-4}(1+(\frac{2\pi\alpha'}{L^2})^2B^2v_*^4)+(\frac{2\pi\alpha'}{L^2})^2(J^t)^2v_*^4}. \tag{4.38}$$

When the second term in the square root dominates and B is small, which is relevant to experimental limit, the ratio becomes $\sigma^{xx}/\sigma^{xy}\sim v_*^{-2}\sim 1/\sigma^{xx}$. Thus the result obtained from the probe calculation mimics the Drude theory in arbitrary dimensions.

When a non-trivial dilaton is introduced, the effective action becomes

$$\tilde{S}_q = -\tilde{\tau}_{\rm eff}\int dtdvd^pxe^{-\phi}\sqrt{-\det(g_{ab}+2\pi\alpha'F_{ab})}. \tag{4.39}$$

The ansatz for the worldvolume gauge fields is still given by (4.24). Therefore the effective action turns out to be

$$\tilde{S}_q = -\tilde{\tau}_{\rm eff}\int dtdvd^pxe^{-\phi}g_{xx}^{\frac{p-2}{2}}\sqrt{GF}, \tag{4.40}$$

where GF is still written in the form of (4.26). The corresponding conserved quantities are given by

$$[-g_{xx}^2 A_0' - (2\pi\alpha')^2 B^2 A_0' + (2\pi\alpha')^2 EBf_y'](\sqrt{GF})^{-1} e^{-\phi} g_{xx}^{\frac{p-2}{2}} = \tilde{C}, \tag{4.41}$$

$$-g_{tt} g_{xx} f_x' (\sqrt{GF})^{-1} e^{-\phi} g_{xx}^{\frac{p-2}{2}} = \tilde{H}, \tag{4.42}$$

$$[-g_{tt} g_{xx} f_y' - (2\pi\alpha')^2 E^2 f_y' + (2\pi\alpha')^2 EBA_0'](\sqrt{GF})^{-1} e^{-\phi} g_{xx}^{\frac{p-2}{2}} = \tilde{M}. \tag{4.43}$$

The on-shell effective action can be written in the following form

$$\tilde{S}_q = \tilde{\tau}_{\text{eff}} \int dv e^{-2\phi} g_{xx}^{p-1} \sqrt{-g_{tt} g_{vv}} \frac{\xi}{\sqrt{\xi\tilde{\eta} - a^2}}, \tag{4.44}$$

where ξ and a are still given by (4.31) and (4.33) with C, H and M replaced by $\tilde{C}, \tilde{H}$ and $\tilde{M}$, while $\tilde{\eta}$ is slightly different from η,

$$\tilde{\eta} = -g_{tt} g_{xx}^p e^{-2\phi} - (2\pi\alpha')^2 [g_{tt}\tilde{C} + g_{xx}(\tilde{H} + \tilde{M})]. \tag{4.45}$$

Finally the conductivity tensor is given by

$$\tilde{\sigma}^{xx} = \frac{1}{1 + (\frac{2\pi\alpha'}{L^2})^2 B^2 v_*^4} \sqrt{(2\pi\alpha')^4 e^{-2\phi_*} \tilde{\tau}_{\text{eff}}^2 (\frac{L}{v_*})^{2p-4} (1 + (\frac{2\pi\alpha'}{L^2})^2 B^2 v_*^4) + (\frac{2\pi\alpha'}{L^2})^2 (J^t)^2 v_*^4}, \tag{4.46}$$

$$\sigma^{xy} = \frac{BJ^t v_*^4}{(\frac{L^2}{2\pi\alpha'})^2 + B^2 v_*^4}. \tag{4.47}$$

It can be easily seen that $\tilde{\sigma}^{xx}$ reduces to σ^{xx} when $\phi = \phi_0$ and σ^{xy} remains invariant.

4.3 Resistivity in different limits

Once we have obtained the conductivity, we may investigate the behavior of the resistivity to see if it exhibits strange metallic behavior, that is, the resistivity has a linear dependence on the temperature T. Since the conductivity contains two terms inside the square root, it is more transparent to study the resistivity in two different limits.

Recall that the conductivity is given by

$$\sigma = \sqrt{(2\pi\alpha')^4 \tau_{\text{eff}}^2 (\frac{L}{v_*})^{2p-4} + (\frac{2\pi\alpha'}{L^2})^2 (J^t)^2 v_*^4}.$$

One limit which is experimentally interesting is large J^t, then the first term is subdominant and the conductivity turns out to be

$$\sigma = \frac{2\pi\alpha'}{L^2} J^t v_*^2. \tag{4.48}$$

Then by combining the fact that $T \propto 1/v_+^z$ we can obtain

$$\rho = \frac{1}{\sigma} \sim \frac{T^{2/z}}{J^t}. \tag{4.49}$$

It seems that the resistivity exhibit a universal behavior in the limit of large charge density, that is, it is proportional to $T^{2/z}$ and is independent of the spacetime dimension. Therefore the conductivity is of the strange metal type in all dimensions for $z = 2$.

The other opposite limit is the zero density limit, which means that the first term in the square root dominates. Then the conductivity is given by

$$\sigma = (2\pi\alpha')\tau_{\text{eff}}(\frac{L}{v_*})^{p-2}. \tag{4.50}$$

When $p = 2$, the conductivity becomes constant, which agrees with the conclusion in [33]. For the $p > 2$ cases, the resistivity becomes

$$\rho \sim T^{-(p-2)/z}, \tag{4.51}$$

where we have used the fact that $T \sim 1/v_+^z$. One can see that for $p > 2$, the linear dependence on the temperature cannot be realized for $z > 0$.

When a non-trivial dilaton is incorporated, the conductivity reads

$$\tilde{\sigma} = \sqrt{(2\pi\alpha')^4 e^{-2\phi_*}\tilde{\tau}_{\text{eff}}^2(\frac{L}{v_*})^{2p-4} + (\frac{2\pi\alpha'}{L^2})^2 (J^t)^2 v_*^4}.$$

One can easily observe that the resistivity is still given by (4.49) in the large charge density limit, that is, the resistivity has a linear dependence on T with $z = 2$ in all dimensions. When the first term dominates, we can assume that $e^{-\phi} \propto 1/v^{\kappa}$, then

$$\rho \sim T^{-\frac{\kappa+p-2}{z}}. \tag{4.52}$$

Therefore the linear dependence on the temperature can be realized by requiring $z = -(\kappa + p - 2)$. In particular, in four-dimensional spacetime with $p = 2$, we can arrive at the linear dependence as long as $z = -\kappa$.

5 DC conductivity revisited and charge diffusion constant

We obtained the DC conductivity in the previous section by introducing an electric field E on the worldvolume of the probe D-branes and making use of Ohm's law $J = \sigma E$. In some sense this approach can be seen as "macroscopic", while the corresponding "microscopic" approach is to calculate the conductivity via Kubo formula. A systematic analysis for calculating the holographic spectral functions for Dp/Dq systems at finite chemical potential and spatial momentum was carried out in [34], where it was found that the conductivity obtained via Kubo formula was in agreement with that obtained in the "macroscopic" approach in the vanishing electric field limit. This can be seen as a non-trivial check of consistency. Holographic spectral functions with non-vanishing electric field strength was investigated in [35], where precise agreement was found once again. Furthermore, conductivity and diffusion constant for Dp/Dq systems were studied in [36], generalizing the formulae derived via the membrane paradigm [37]. In this section we will revisit the conductivity following [34] and we will find that the result agrees with that obtained in previous section. The charge diffusion constant in such asymptotically Lifshitz backgrounds will also be discussed.

5.1 Calculating DC conductivity via Kubo formula

Consider the following fluctuations of the worldvolume gauge field

$$A_a \;\to\; \delta_{a0}A_0(r) + \epsilon e^{-i(\omega x^0 - q x^1)}\mathcal{A}_a(r), \tag{5.1}$$

the DBI Lagrangian can be expanded in powers of ϵ,

$$\mathcal{L}_{\text{DBI}} = \mathcal{L}_0 + \epsilon\mathcal{L}_1 + \epsilon^2\mathcal{L}_2 + \cdots\,. \tag{5.2}$$

Notice that $\mathcal{L}_1$ vanishes identically upon imposing the equations of motion for the background fields. We shall not present the detailed analysis for the fluctuations but just exhibit the equations of motion,

$$\mathcal{A}_\perp'' + \partial_r \log[\sqrt{-\gamma}\gamma^{rr}\gamma^{ii}]\mathcal{A}_\perp' - \frac{\omega^2\gamma^{00} + q^2\gamma^{ii}}{\gamma^{rr}}\mathcal{A}_\perp = 0, \tag{5.3}$$

$$\mathcal{A}_0'' + \partial_r \log[\sqrt{-\gamma}\gamma^{rr}\gamma^{00}]\mathcal{A}_0' - q\frac{\gamma^{ii}}{\gamma^{rr}}(q\mathcal{A}_0 + \omega\mathcal{A}_1) = 0, \tag{5.4}$$

$$\mathcal{A}_1'' + \partial_r \log[\sqrt{-\gamma}\gamma^{rr}\gamma^{ii}]\mathcal{A}_1' - \omega\frac{\gamma^{00}}{\gamma^{rr}}(q\mathcal{A}_0 + \omega\mathcal{A}_1) = 0, \tag{5.5}$$

as well as the constraint from the gauge choice $\mathcal{A}_r = 0$

$$-\omega\gamma^{00}\mathcal{A}_0' + q\gamma^{ii}\mathcal{A}_1' = 0, \tag{5.6}$$

for details see Appendix A of [34]. Here we adopt the notations of [34],

$$F_{ab} = \partial_a A_b - \partial_b A_a \equiv F_{ab}^{(0)} + \epsilon F_{ab}^{(1)}, \tag{5.7}$$

$$\gamma_{ab} = g_{ab} + F_{ab}^{(0)}, \quad \gamma^{ab} = (\gamma_{ab})^{-1}. \tag{5.8}$$

Notice that the equation of the transverse fluctuations $\mathcal{A}_\perp$ is decoupled from others. Another point is that (5.4), (5.5) and (5.6) are not linearly independent, as one can derive (5.5) by combining (5.4) and (5.6).

One may expect that we can perform the hydrodynamic expansions for $\mathcal{A}_a$ and try to solve the corresponding equations, then the retarded correlation functions can be obtained following the standard procedure [38]. But it is quite difficult to solve the relevant equations for the longitudinal fluctuations indeed. However, the DC conductivity can be determined from the retarded correlation function of the transverse modes via Kubo formula

$$\sigma = \lim_{\omega\to 0} \mathrm{Im}\frac{1}{\omega}G_\perp^R(\omega, q=0). \tag{5.9}$$

Then it is sufficient to work with $q=0$ to find the conductivity, which was done for AdS case in [39]. Under this condition the equation of motion for $\mathcal{A}_\perp$ can be simplified as

$$\mathcal{A}_\perp'' + \partial_r \log[\sqrt{-\gamma}\gamma^{rr}\gamma^{ii}]\mathcal{A}_\perp' - \frac{\omega^2\gamma^{00}}{\gamma^{rr}}\mathcal{A}_\perp = 0, \tag{5.10}$$

which allows us to find analytic solutions in the hydrodynamic expansions. For general Dp/Dq systems the final result was given in [34]

$$\sigma = \mathcal{N}\sqrt{-\gamma\gamma^{00}\gamma^{rr}}\gamma^{ii}\big|_{r\to r_H}, \tag{5.11}$$

where $\mathcal{N} = (2\pi\alpha')^2\tau_{\text{eff}}$ and r_H denotes the horizon. Notice that we have absorbed the constant dilaton into $\mathcal{N}$.

For our specific model, the components of γ_{ab} are given by

$$\gamma_{00} = g_{00} = -\frac{f(v)}{v^{2z}}, \quad \gamma_{vv} = \frac{1}{v^2 f(v)}, \quad \gamma_{ii} = g_{ii} = \frac{1}{v^2},$$
$$\gamma_{0v} = -2\pi\alpha' A_0', \quad \gamma^{00} = \frac{\gamma_{vv}}{\gamma_{00}\gamma_{vv} + \gamma_{0v}^2}, \quad \gamma_{vv} = \frac{\gamma_{00}}{\gamma_{00}\gamma_{vv} + \gamma_{0v}^2}. \tag{5.12}$$

One can obtain the following result for A_0' by setting $E = H = 0$ in (4.6),

$$A_0'^2 = -\frac{C^2\gamma_{00}\gamma_{vv}}{\gamma_{ii}^p + (2\pi\alpha')^2 C^2}. \tag{5.13}$$

Finally by combining (5.11), (5.12) and (5.13), we can arrive at

$$\sigma = \sqrt{(2\pi\alpha')^4 \tau_{\text{eff}}^2 (\frac{L}{v_+})^{2p-4} + (\frac{2\pi\alpha'}{L^2})^2 (J^t)^2 v_+^4}, \tag{5.14}$$

where we have used the relation $J^t = (2\pi\alpha')^2 \tau_{\text{eff}} C$. One can compare this result with (4.18). From (4.14) we can see that in the limit of $E \to 0$, $v_* \to v_+$. Then (5.14) is in agreement of (4.18), which also provides a non-trivial check of consistency. Furthermore, in the zero density limit $J^t \to 0$, the conductivity is given by

$$\sigma = \tau_{\text{eff}} (2\pi\alpha')^2 \frac{L^{p-2}}{v_+^{p-2}}, \tag{5.15}$$

which agrees with the result obtained in [40].

When a non-trivial dilaton is considered, the conductivity can be written as

$$\tilde{\sigma} = \tilde{\mathcal{N}} e^{-\phi} \sqrt{-\gamma\gamma^{00}\gamma^{vv}} \gamma^{ii} \Big|_{v \to v_+}, \tag{5.16}$$

where $\tilde{\mathcal{N}} = (2\pi\alpha')^2 \tilde{\tau}_{\text{eff}}$. Now the solution for A_0' is given by

$$A_0'^2 = -\frac{C^2\gamma_{00}\gamma_{vv}}{e^{-2\phi}\gamma_{ii}^p + (2\pi\alpha')^2 C^2}. \tag{5.17}$$

Then the conductivity reads

$$\tilde{\sigma} = \sqrt{e^{-2\phi_+} (2\pi\alpha')^4 \tilde{\tau}_{\text{eff}}^2 (\frac{L}{v_+})^{2p-4} + (\frac{2\pi\alpha'}{L^2})^2 (J^t)^2 v_+^4}, \tag{5.18}$$

which agrees with (4.23) once again in the limit of $E \to 0$. Notice that (5.18) reduces to (5.14) when the dilaton becomes trivial $\phi = \phi_0$.

5.2 Charge diffusion constant

Generally speaking, the charge diffusion constant can be read off from the denominator of the tt-component of the longitudinal correlator. However, since it is quite difficult to solve the equations of motion for longitudinal fluctuations, even in the hydrodynamic expansions, we cannot expect that the charge diffusion constant can be obtained in that way. An alternative formula for estimating the charge diffusion constant was derived in [37] from the membrane paradigm. Such a formula was generalized to Dp/Dq systems in [36] by considering the equation of motion for the gauge invariant perturbations $E_\parallel = q\mathcal{A}_0 + \omega\mathcal{A}_1$. At zero baryon density the corresponding equation of motion is

$$E_\parallel'' + \partial_r \log[\frac{e^{-\phi}\sqrt{-\gamma}\gamma^{ii}\gamma^{rr}}{\omega^2 + q^2\frac{\gamma^{ii}}{\gamma^{00}}}]E_\parallel' - \frac{\omega^2\gamma^{00} + q^2\gamma^{ii}}{\gamma^{rr}}E_\parallel = 0. \tag{5.19}$$

Assuming a dispersion relation of the form $\omega = -iDq^2 + \cdots$, the natural hydrodynamic scaling is given by $\omega \to \lambda^2\omega, q \to \lambda q$. Then the near horizon expansion can be written as

$$E_\parallel(r) = F(r)^{-i\frac{\lambda^2\omega}{4\pi T}}(E^{(0)}_{\parallel,\mathrm{reg}} + \lambda^2 E^{(2)}_{\parallel,\mathrm{reg}} + \cdots) = E^{(0)}_\parallel + \lambda^2 E^{(2)}_\parallel + \cdots \tag{5.20}$$

where $F(r) = (r - r_H)F_{\mathrm{reg}}(r)$ and the subscript "reg" stands for the regular part of the corresponding function. The solution at lowest order is given by

$$E^{(0)}_\parallel = 1 - iC_0\frac{q^2}{2\pi\omega T}\int_{r_H}^r \frac{dr}{e^{-\phi}\sqrt{-\gamma}\gamma^{00}\gamma^{rr}}, \quad C_0 = -\frac{1}{2}e^{-\phi}\sqrt{-\gamma}\gamma^{00}\gamma^{rr}F'\big|_{r\to r_H}. \tag{5.21}$$

Notice that the retarded correlation function $G^R_\parallel \sim \lim_{r\to r_B} E_\parallel'(r)/E_\parallel(r_B)$ where r_B denotes the boundary. Then the dispersion relation comes from requiring $E_\parallel(r_B) = 0$. Finally the charge diffusion constant is determined as

$$D_0 = e^{-\phi}\sqrt{-\gamma\gamma^{00}\gamma^{rr}}\gamma^{ii}\big|_{r_+}\int_{r_+}^{r_B}\frac{dr}{e^{-\phi}\sqrt{-\gamma}\gamma^{00}\gamma^{rr}}, \tag{5.22}$$

which reduces to the one in [37] when $A_0 = 0$. The charge diffusion constant for massive charge carriers at finite baryon density is more complicated [36]. However, for massless charge carriers at finite density the charge diffusion constant is still given by (5.22).

Let us return to our concrete example. Since the integral in (5.22) involves the dilaton and we do not give its explicit form, we consider the trivial dilaton case instead. After substituting (5.12) and (5.13) into (5.22), we can obtain

$$D_0 = \frac{1}{(p-z)v_+^{z-2}}\sqrt{1 + (\frac{2\pi\alpha'}{L^p})^2C^2v_+^{2p}}\,{}_2F_1[\frac{p-z}{2p}, \frac{3}{2}; \frac{3}{2} - \frac{z}{2p}; -(\frac{2\pi\alpha'}{L^p})^2C^2v_+^{2p}]. \tag{5.23}$$

It can be seen that the above result reduces to the one obtained in [40] when the charge density parameter $C = 0$. Another point is that the charge diffusion constant becomes divergent when $z = p$, which was also observed in [40] for zero density case. As a check of consistency, we can choose one specific example $z = 1, p = 3$ and (5.23) becomes

$$D_0 = \frac{1}{2\pi T}\sqrt{1+d^2}{}_2F_1[\frac{1}{3},\frac{3}{2};\frac{4}{3};-d^2], \quad d = 2\pi\alpha' C v_+^3/L^6, \tag{5.24}$$

which agrees with the result for D3/D7 in [41].

One may wonder if the Einstein relation still holds for our system. The charge susceptibility is determined by

$$\chi = \frac{\partial n_q}{\partial \mu}|_T, \tag{5.25}$$

where $n_q = \delta S_q/\delta A_0'$ is the charge density and μ denotes the chemical potential. Then

$$\chi = (\int_{r_H}^{r_B} \frac{dA_0'}{dn_q} dr)^{-1}. \tag{5.26}$$

It has been observed in [36] that for massless charge carriers the charge susceptibility has a simple form

$$\chi = \mathcal{N}(\int_{r_H}^{r_B} \frac{1}{e^{-\phi}\sqrt{-\gamma}\gamma^{00}\gamma^{rr}} dr)^{-1}. \tag{5.27}$$

Therefore by combining (5.22) and (5.27), we can conclude that the Einstein relation $D_0 = \sigma/\chi$ holds for massless charge carriers.

6 Remarks on massive charge carriers

In this section we will give some remarks on the properties of massive charge carriers. In general, the properties for massive charge carriers exhibit qualitatively similar behavior as their massless counterparts, so we will just present the main results in brief.

6.1 Thermodynamics

For massive charge carriers the embedding profile of the probe Dq-brane is described by a non-trivial function $\theta(r)$. Then the induced metric on the Dq-brane is given by

$$ds^2_{D_q} = -r^{2z}f(r)dt^2 + (\frac{1}{r^2 f(r)} + \theta(r)'^2)dr^2 + r^2 d\vec{x}_p^2. \tag{6.1}$$

The effective action takes the following form for the trivial dilaton case

$$S_q = -\tau_{\text{eff}} \int dr r^p \cos^n \theta \sqrt{r^{2z-2} + r^2 f(r) \theta'^2 - A_0'^2}. \tag{6.2}$$

Similarly the solution for A_0' is

$$A_0' = \sqrt{\frac{r^{2z-2} + r^2 f(r) \theta'^2}{1 + \frac{r^{2p} \cos^{2n} \theta}{\tilde{d}^2}}}, \tag{6.3}$$

where $\tilde{d} = \rho/\tau_{\text{eff}}$ and $\rho = \delta \mathcal{L}_q / \delta A_0'$. It has been pointed out in [26] that the equation of motion for the embedding profile $\theta(r)$ is not analytically solvable except in the limit of zero temperature.

We can study the free energy in canonical ensemble by performing the Legendre transformation to the action

$$\Omega = \tilde{d} \tau_{\text{eff}} \int_{r_+}^{\infty} dr \sqrt{r^{2z-2} f(r) + r^2 f(r) \theta'^2} \sqrt{1 + \frac{r^{2p} \cos^{2n} \theta}{\tilde{d}^2}}. \tag{6.4}$$

Following [27], to investigate the low temperature behavior of the free energy we may perform a low-temperature expansion around $r_+ = 0$,

$$\Omega = \Omega(r_+)\big|_{r_+=0} + (\frac{\partial \Omega}{\partial r_+})\big|_{r_+=0} r_+ + \mathcal{O}(r_+^2). \tag{6.5}$$

After carefully evaluating the behavior of Ω at the horizon and at the boundary, we can obtain the expansion similar to that in [27]

$$\Omega = \Omega_0 + \rho r_+ + \mathcal{O}(r_+^2). \tag{6.6}$$

One crucial point that is different from [27] is that the free energy contributes to the entropy even at zero temperature, which is the constant term given in (3.15). Therefore the specific heat is dominated by the subleading term, which is similar to the cases discussed in [26]. For the case of a non-trivial dilaton the conclusion is similar.

6.2 Conductivity and drag force

By assuming the same configuration of the worldvolume gauge fields and following the same steps presented in section 4, we can arrive the following results for the DC conductivity and the DC Hall conductivity for the case of a trivial dilaton

$$\sigma = \sqrt{(2\pi\alpha')^4 \tau_{\text{eff}}^2 \cos^{2n} \theta_* (\frac{L}{v_*})^{2p-4} + (\frac{2\pi\alpha'}{L^2})^2 (J^t)^2 v_*^4}, \tag{6.7}$$

$$\sigma^{xx} = \frac{1}{1+(\frac{2\pi\alpha'}{L^2})^2 B^2 v_*^4}\sqrt{(2\pi\alpha')^4\tau_{\text{eff}}^2\cos^{2n}\theta_*(\frac{L}{v_*})^{2p-4}(1+(\frac{2\pi\alpha'}{L^2})^2B^2v_*^4)+(\frac{2\pi\alpha'}{L^2})^2(J^t)^2v_*^4},$$
$$\sigma^{xy} = \frac{BJ^tv_*^4}{(\frac{L^2}{2\pi\alpha'})^2+B^2v_*^4}. \tag{6.8}$$

Notice that the quantities ξ and a are still given by (4.31) and (4.33), while the expression for η receives some modifications

$$\eta = -g_{tt}g_{xx}^p\cos^{2n}\theta-(2\pi\alpha')^2[g_{tt}C^2+g_{xx}(H^2+M^2)]. \tag{6.9}$$

Here v_* is still determined by $\xi(v_*)=0$. For the case of a non-trivial dilaton,

$$\tilde{\sigma} = \sqrt{(2\pi\alpha')^4e^{-2\phi_*}\tilde{\tau}_{\text{eff}}^2\cos^{2n}\theta_*(\frac{L}{v_*})^{2p-4}+(\frac{2\pi\alpha'}{L^2})^2(J^t)^2v_*^4}, \tag{6.10}$$

$$\tilde{\sigma}^{xx} = \frac{1}{1+(\frac{2\pi\alpha'}{L^2})^2 B^2 v_*^4}\sqrt{(2\pi\alpha')^4e^{-2\phi_*}\tilde{\tau}_{\text{eff}}^2\cos^{2n}\theta_*(\frac{L}{v_*})^{2p-4}(1+(\frac{2\pi\alpha'}{L^2})^2B^2v_*^4)+(\frac{2\pi\alpha'}{L^2})^2(J^t)^2v_*^4},$$
$$\sigma^{xy} = \frac{BJ^tv_*^4}{(\frac{L^2}{2\pi\alpha'})^2+B^2v_*^4}. \tag{6.11}$$

In this case ξ and a still remain invariant, while $\tilde{\eta}$ becomes

$$\tilde{\eta} = -g_{tt}g_{xx}^pe^{-2\phi}\cos^{2n}\theta-(2\pi\alpha')^2[g_{tt}C^2+g_{xx}(H^2+M^2)]. \tag{6.12}$$

It can be seen that the conductivity for massive charge carriers exhibits qualitatively similar behavior as their massless counterparts, up to a factor of $\cos^{2n}\theta_*$ in the first term inside the square root. Furthermore, the DC Hall conductivity remains invariant for all cases. Therefore the resistivity in different limits also exhibits similar behavior as the massless case.

From a "microscopic" point of view, the conductivity is still given by (5.11) for the case of trivial dilaton or (5.16) for the case of non-trivial dilaton, which gives

$$\sigma = \sqrt{(2\pi\alpha')^4\tau_{\text{eff}}^2(\frac{L}{v_+})^{2p-4}\cos^{2n}\theta_+ +(\frac{2\pi\alpha'}{L^2})^2(J^t)^2v_+^4}, \tag{6.13}$$

$$\tilde{\sigma} = \sqrt{(2\pi\alpha')^4e^{-2\phi_+}\tilde{\tau}_{\text{eff}}^2(\frac{L}{v_+})^{2p-4}\cos^{2n}\theta_+ +(\frac{2\pi\alpha'}{L^2})^2(J^t)^2v_+^4}. \tag{6.14}$$

We can find agreement with the "macroscopic" results in the $E\to 0$ limit once again.

In the large mass limit, the relations between the conductivity and/or Hall conductivity and the drag force were clarified in [31] and [32]. As shown in [32], in the large mass limit the flavor excitations are expected to be described as a collection of quasiparticles, obeying the following equation of motion

$$\frac{d\vec{p}}{dt} = \vec{E} + \vec{v} \times \vec{B} - \vec{F}_{\text{drag}}, \tag{6.15}$$

where v stands for the quasiparticle velocity. In the steady state $d\vec{p}/dt = 0$, we have

$$F_{\text{drag}} = \sqrt{E^2 + v^2 B^2 + 2\vec{E} \cdot (\vec{v} \times \vec{B})}. \tag{6.16}$$

The thermal pair creation should be suppressed in the large mass limit, hence

$$J^x = J^t v_x, \quad J^y = J^t v_y. \tag{6.17}$$

By setting $\chi(v_*) = 0$ we can obtain $v^2 = |g_{tt}|/g_{xx}$, where the first term has been suppressed in the large mass limit. Setting $\xi = a = 0$ leads to the following results

$$E^2 = \frac{1}{(2\pi\alpha')^2} g_{xx}^2 v^2 + v^2 B^2, \quad v_y = -v^2 \frac{B}{E}. \tag{6.18}$$

Thus $2\vec{E} \cdot (\vec{v} \times \vec{B}) = -2B^2 v^2$, which determines the drag force

$$F_{\text{drag}} = \frac{1}{2\pi\alpha'} g_{xx}(v_*) v. \tag{6.19}$$

One can see that this is precisely the formula for the drag force in [42, 43, 44].

One can also obtain the conductivity tensor via the drag force. By setting $\xi = 0$ we can obtain

$$f(v_*) = \frac{(2\pi\alpha')^2 E^2 v_*^{2z+2}}{1 + (2\pi\alpha')^2 B^2 v_*^4}. \tag{6.20}$$

Then

$$v^2 = -\frac{g_{tt}}{g_{xx}} = \frac{(2\pi\alpha')^2 E^2 v_*^4}{1 + (2\pi\alpha')^2 B^2 v_*^4}. \tag{6.21}$$

The components of the speed are given by

$$\begin{aligned} v_y &= -v^2 \frac{B}{E} = -\frac{(2\pi\alpha')^2 E B v_*^4}{1 + (2\pi\alpha')^2 B^2 v_*^4}, \\ v_x &= \sqrt{v^2 - v_y^2} = \frac{2\pi\alpha' E v_*^2}{1 + (2\pi\alpha')^2 B^2 v_*^4}. \end{aligned} \tag{6.22}$$

Finally, by combining $J^i = J^t v_i$ and $J^i = \sigma^{ij}E_j, i,j = x,y$, one can arrive at

$$\sigma^{xx} = \frac{\frac{2\pi\alpha'}{L^2}J^t v_*^2}{1+(\frac{2\pi\alpha'}{L^2})^2 B^2 v_*^4}, \quad \sigma^{xy} = \frac{BJ^t v_*^4}{(\frac{L^2}{2\pi\alpha'})^2 + B^2 v_*^4}. \tag{6.23}$$

One can check that these results agree with previous ones in the large mass limit. The relations between drag force and DC conductivity were discussed in [45] and here we generalize it to the case of conductivity tensor.

6.3 Charge diffusion constant

The charge diffusion constant becomes more subtle for massive charge carriers, as studied in [36]. For completeness we just list the result here, for details see [36].

The charge diffusion constant for massive charge carriers with trivial dilaton is given by

$$D = \frac{D_0}{1+\frac{n_q}{2\pi\alpha'\tau_{\text{eff}}}\int\frac{\Delta\tilde{\Psi}'_{(0)}+\Xi\tilde{\Psi}_{(0)}}{\sqrt{-\gamma}\gamma^{00}\gamma^{rr}}}, \tag{6.24}$$

where D_0 is given by (5.22) and

$$\Delta = \gamma^{rr}\psi' g_{\psi\psi}, \quad \Xi = \frac{1}{2}(\gamma^{rr}\psi'^2 g_{\psi\psi,\psi} - n\gamma^{\theta\theta}\gamma_{\theta,\psi}) \tag{6.25}$$

with $\psi \equiv \sin\theta$. $\tilde{\Psi}_{(0)}$ is related to the zeroth order expansion of the fluctuation of the embedding profile by $\tilde{\Psi}_{(0)} = q\Psi_{(0)}$. For the case of a non-trivial dilaton we just replace τ_{eff} by $\tilde{\tau}_{\text{eff}}$. It can be easily seen that $\Delta = \Xi = 0$ and $D = D_0$ for massless charge carriers.

The charge susceptibility also receives modifications due to the fact that ψ also depends on n_q. For the case of non-trivial dilaton, it is given by

$$\chi = (2\pi\alpha')^2\tilde{\tau}_{\text{eff}}\Big(\int_{r_+}^{r_B}\frac{dr}{e^{-\phi}\sqrt{-\gamma}\gamma^{00}\gamma^{rr}}[1+n_q(\Delta\frac{\partial\psi'}{\partial n_q}+\Xi\frac{\partial\psi}{\partial n_q})]\Big)^{-1}. \tag{6.26}$$

For the case of trivial dilaton one can replace $\tilde{\tau}_{\text{eff}}$ by τ_{eff} and $e^{-\phi}$ by $e^{-\phi_0}$. This expression reduces to the one given in (5.27) for massless charge carriers where $\Delta = \Xi = 0$. The Einstein relation for massive charge carriers should be checked numerically and it was verified to be true for D3/D7 system in [36].

7 Summary and discussion

We study several properties of holographic strange metals in this note. The dual bulk spacetime is $p+2$-dimensional Lifshitz black holes and the role of charge carriers is played by probe Dq-branes. The cases of trivial dilaton and non-trivial dilaton are discussed respectively. We calculate the free energy density and chemical potential analytically for massless charge carriers, expressing the results in terms of hypergeometric functions. The entropy density and heat capacity at low temperature are also obtained. We find that the heat capacity behaves like a gas of free bosons at $z = 2$ and like a gas of fermions at $z = 2p$. It may indicate a new type of quantum liquid for other cases. The speed of sound at low temperature is also evaluated.

We calculate the DC conductivity and DC Hall conductivity in $p + 2$-dimensions, generalizing the four-dimensional results obtained in [15]. There are always two terms inside the square root of DC conductivity, one of which can be interpreted as describing thermal pair production and is suppressed in the large mass limit. The other term is proportional to the charge density J^t and takes a universal form in all dimensions. Furthermore, the DC Hall conductivity also takes a universal form, which is independent of p. We discuss the resistivity in different limits and find that at large charge density, the resistivity is linear dependent on temperature when $z = 2$, while for the case of trivial dilaton, the linear dependence cannot be realized for $p > 2$ with $z > 0$ at vanishing charge density. When a non-trivial dilaton is introduced, such a linear dependence can still be realized by adjusting the parameters even at $p = 2$.

As a check of consistency, we apply the formulae for conductivity in [34] to our backgrounds. Such formulae were derived via Kubo formula then they could be seen as "microscopic" results. We find that the "microscopic" results are in agreement with the "macroscopic" results obtained in section 4. We also obtain an analytic result for the charge diffusion constant, which agrees with the result for D3/D7 in specific limit. The Einstein relation explicitly holds for massless charge carriers. We also give some remarks on the corresponding properties of massive charge carriers. In particular, the conductivity tensor can be reproduced via the drag force calculations in the large mass limit.

There are several interesting directions which are worth investigating. For massive charge carriers the fluctuations of the embedding profile couple to the longitudinal fluctuations of

the worldvolume gauge fields, which results in highly complicated differential equations. Recently a framework for calculating holographic Green's functions from general bilinear actions and fields obeying coupled differential equations in the bulk was proposed in [46, 47]. Then it would be interesting to classify the vector and scalar fluctuations and calculate the Green's functions for our system by employing their method. Another point is to study the fermionic properties of the background. In particular, it would be desirable to obtain the Green's functions for the fermions, following [48, 49, 14]. We expect to study such fascinating topics in the future.

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